

# EXPERIMENTAL UNCERTAINTY SURVEY AND ASSESSMENT

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## 1.0 SUMMARY

As detailed in the Statement of Work for this research effort, the principal investigator was to:

(1) survey the experimental efforts and facilities within the NASA/MSFC Propulsion Laboratory,

(2) examine the uncertainty methodologies, approaches, and techniques currently used within the laboratory and those currently required of contractors in propulsion research,

(3) assess strengths and weaknesses of current uncertainty approaches and provide recommendations as appropriate, and

(4) provide specific recommendations for improvements in error analysis and the management of experimental uncertainty to aid in (A) the validation of power balance and internal flow codes based on data from the Technology Test Bed (TTB) engine and (B) the assessment of the rated performance of flight Space Shuttle Main Engines (SSME's) based on acceptance test and flight data analysis.

During a meeting on August 23, 1991 with Jerry Redus, David Seymour, Margie Zoladz and Charles Martin of the Propulsion Laboratory, it was decided that the areas of primary initial interest were the uncertainties associated with specific impulse determination in SSME ground testing (including the TTB program) and in Solid Rocket Motor (SRM) ground testing. Also of interest were the uncertainties associated with scale-up of the Space Transportation Main Engine (STME) to the final configuration and uncertainty

considerations when comparing SSME specific impulse values from ground tests and from flight data.

After surveying documents on SSME design and operation and beginning to examine the voluminous materials from SSME Performance Review Meetings from July 1990 forward, the PI decided to concentrate the remainder of this one man-month effort primarily on assessing the uncertainties associated with SSME Acceptance Tests at Stennis Space Center (SSC). The results of this assessment are described in Section 2 of this report.

Specific conclusions and recommendations are presented in Section 3. These can be summarized by stating that it is necessary to consider separately the random (precision) and fixed (bias) components of uncertainty when evaluating experimental results and comparing results from different tests. It is also necessary to consider possible correlated bias error effects, particularly in the TTB program when results from different tests using the same test stand and base engine are compared.

## 2.0 SSME GROUND TEST UNCERTAINTY ASSESSMENT

### 2.1 Uncertainty Analysis Background

An experiment is performed to answer a question or to find the solution to a problem. Designate the true answer to the question or true solution to the problem as  $r_{\text{true}}$ . The result which is obtained from the experiment is  $r$ , a flawed estimate of  $r_{\text{true}}$ . To properly interpret the meaning of the result  $r$ , an estimate of  $\pm U_x$ , the interval around  $r$  in which we are 95% confident that  $r_{\text{true}}$  lies.

The terms necessary to determine the 95% confidence interval covering the true result are defined below:

Precision Limit,  $P_r$ . The  $\pm P_r$  interval about a result is the band within which the mean result,  $\mu$ , would fall 95% of the time if the experiment were repeated many times under the same conditions using the same equipment. The precision limit is a result of the scatter (or lack of repeatability) caused by random errors and unsteadiness.

Bias Limit,  $B_r$ . The bias limit is an estimate of the magnitude of the fixed, constant errors. When the true bias error in a result is defined as  $\beta$ , the quantity  $B_r$  is the experimenter's 95% confidence estimate such that

$$|\beta| \leq B_r.$$

Uncertainty,  $U_r$ . The  $\pm U_r$  interval about the result is the band within which the experimenter is 95% confident the true value of the result lies. The 95% confidence uncertainty is calculated from [1] as

$$U_r = [B_r^2 + P_r^2]^{1/2} \quad (1)$$

If an experiment has been repeated a number of times so that M previous results are available (such as multiple SSME tests on the same test stand), then the best estimate of the precision limit to associate with another similar result would be  $P_r$  calculated as

$$P_r = t S_r \quad (2)$$

where t is the 95% confidence level value of the Student's t distribution for  $v = M - 1$  degrees of freedom.  $S_r$  is the precision index (sample standard deviation) of the set of M results and is defined by

$$S_r = \left[ \frac{1}{M-1} \sum_{k=1}^M (r_k - \bar{r})^2 \right]^{1/2} \quad (3)$$

where  $\bar{r}$  is the mean of the M  $r_k$ 's.

The bias limit,  $B_r$ , is the experimenter's 95% confidence estimate of the magnitude of the fixed error in the result. When the result r is determined from measured values of J variables

$$r = r(X_1, X_2, \dots, X_J) \quad (4)$$

then the bias limit of the result is related to the bias limits  $B_i$  of the measurements of the separate variables  $X_i$  by

$$B_r^2 = \sum_{i=1}^J \left[ \theta_i^2 B_i^2 + \sum_{k=1}^J \theta_i \theta_k \rho_{ik} B_i B_k (1 - \delta_{ik}) \right] \quad (5)$$

where

$$\theta_i = \frac{\partial r}{\partial X_i} \quad (6)$$

$\rho_{ik}$  is the correlation coefficient for the biases in the measurements of  $X_i$  and  $X_k$ , and  $\delta_{ik}$  is the Kronecker delta. The bias limits  $B_i$  are estimates at 95% confidence of the magnitude of the fixed errors in the measurements of the separate variables  $X_i$ .

In practice the correlated biases are usually handled by making the approximation

$$\rho_{ik} B_i B_k = (1) B_i' B_k' \quad (7)$$

so that

$$B_r^2 = \sum_{i=1}^J \left[ \theta_i^2 B_i^2 + \sum_{k=1}^J \theta_i \theta_k B_i' B_k' (1 - \delta_{ik}) \right] \quad (8)$$

where  $B'_i$  and  $B'_k$  are the portions of biases in measurements of variables  $X_i$  and  $X_k$  that arise from the same sources and are presumed to be perfectly correlated (Coleman and Steele, [2]).

Correlated bias errors are those that are not independent of each other. There has been very little discussion in the engineering literature of these concepts or their application. The ANSI/ASME Standard on Measurement Uncertainty [1] mentions correlated bias errors only in one of the examples presented. Coleman and Steele presented a derivation of the propagation equation for bias errors including the effects of correlated elemental bias sources [3] and also discussions of the approximation of such terms in practical applications [2].

## 2.2 SSME Specific Impulse Uncertainty Analysis

The SSME vacuum specific impulse  $I$  is determined from

$$I = F_a / W_a \quad (9)$$

where  $F_a$  is the adjusted thrust and  $W_a$  is the adjusted mass flow rate of the propellants. For tests using SSC test stand A-2, these are determined from

$$F_a = F_{lc} - F_{zs} + F_{pw} + F_o + F_f + F_{rb} + F_{undif} + F_{dif} + F_{clam} + F_{base} \quad (10)$$

and

$$W_a = \rho_o Q_o + \rho_f Q_f + W_{orp} + W_{frp} \quad (11)$$



The specific impulse is thus a function of 16 variables.

$F_{lc}$  is the sum of the forces measured by the 3 load cells, and it is considered as one variable in the uncertainty analysis because the calibration procedure effectively calibrates the sum of the outputs of the 3 load cells rather than treating them individually.  $F_{zs}$  is the sum of the zero shifts measured using the 3 load cells prior to engine startup. The next four variables in Equation (10) --  $F_{pw}$ ,  $F_o$ ,  $F_f$ , and  $F_{rb}$  -- are corrections for propellant weight, oxygen and fuel inlet momentum gains, and for the reaction beam effects, respectively. The final four variables in Equation (10) are corrections to vacuum conditions, with  $F_{undif}$ ,  $F_{dif}$ ,  $F_{clam}$ , and  $F_{base}$  being adjustments for the undiffused exit area, the diffused exit area, the clamshell seal area, and the atmospheric base pressure, respectively.

In Equation (11),  $\rho_o$  and  $\rho_f$  are the oxygen and fuel densities at the volumetric flowmeters which measure  $Q_o$  and  $Q_f$ , and  $W_{orp}$  and  $W_{frp}$  are the oxygen and fuel repressurization mass flow rates.

The bias limit of the experimental result,  $I$ , and the precision limit of the experimental result should be considered separately.

### 2.2.1 Specific impulse bias limit estimation

So that specific numerical magnitudes can be investigated, SSME Ground Test A2-542 was chosen as a typical "nominal" test, and the results from that test are given in Figure 1.

**FIGURE 1: SSME SPECIFIC IMPULSE  
DETERMINATION FOR TEST A2-542**

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$$I = F_a / W_a$$

$$F_a = F_{lc} - F_{zs} + F_{pw} + F_o + F_f + F_{rb} + F_{undif} + F_{dif} + F_{clam} + F_{base}$$

$$W_a = \rho_o Q_o + \rho_f Q_f + W_{orp} + W_{frp}$$

**FOR TEST A2-542**

$$\begin{aligned} F_a &= 489,665 \text{ lbf} \\ W_a &= 1,085 \text{ lbm/sec} \\ I &= 451.2 \text{ sec} \end{aligned}$$

$F_{lc}$	451,149	lbf
$F_{zs}$	3,345	lbf
$F_{pw}$	77	lbf
$F_o$	0	lbf
$F_f$	-2,964	lbf
$F_{rb}$	291	lbf
$F_{undif}$	21,172	lbf
$F_{dif}$	13,499	lbf
$F_{clam}$	3,332	lbf
$F_{base}$	28	lbf

$\rho_o$	70.8	lbm/ft <sup>3</sup>
$Q_o$	13.2	ft <sup>3</sup> /sec
$\rho_f$	4.4	lbm/ft <sup>3</sup>
$Q_f$	35.4	ft <sup>3</sup> /sec
$W_{orp}$	-1.6	lbm/sec
$W_{frp}$	-0.7	lbm/sec

In considering the result from a single test, there are no apparent correlated biases of significance. Application of the bias limit propagation expression (Equation (8)) to Equations (9), (10), and (11) and algebraically manipulating the resulting expression into its simplest form yields the equation shown in Figure 2. Also shown in Figure 2 are the values of each of the individual bias limit terms assuming the bias limit for each of the 16 variables is 1%. While we know that this certainly is not so, this view allows us to see which variables are most important from an uncertainty standpoint when all uncertainties are equal. It is apparent that, under these assumptions, the bias errors in the load cell measurements, in the oxygen volumetric flowrate measurement, and in the oxygen density value are the most dominant, followed by the bias errors in the fuel volumetric flowrate and density and the corrections for the undiffused and diffused exit areas. These results identified the variables which should be concentrated upon in this research effort.

A similar presentation is shown in Figure 3, except that in this case estimates for the bias limits identified as being of greatest significance were more carefully made. The bias limits shown in this figure are thought to be good "ballpark" estimates and result in a value of  $B_I$  of 0.33% (or 1.5 seconds). The bias limit for the thrust measured by the load cells,  $B_{Flc}$ , was estimated using the NIST stated accuracy of the calibration standard ( $\approx 0.04\%$  of the full scale of 500,000 lbf) and discussions of the calibration procedure with Rocketdyne personnel at SSC. This author thinks that the 0.08% value in Figure 3 is the lowest estimate that can be justified and is probably on the low side.

**FIGURE 2: SPECIFIC IMPULSE BIAS LIMIT FOR  
TEST A2-542 IF ALL B's 1% ( $B_i = 1.5\%$  or 7 sec)**

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$(B_i / I)^2 =$	<u>Value (<math>\times 10^{-6}</math>)</u>
$(B_{Flc} / F_a)^2$	85.
$+ (B_{Fzs} / F_a)^2$	0.004
$+ (B_{Fpw} / F_a)^2$	0.000002
$+ (B_{Fo} / F_a)^2$	0
$+ (B_{Ff} / F_a)^2$	0.003
$+ (B_{Frb} / F_a)^2$	0.00004
$+ (B_{Fundif} / F_a)^2$	0.19
$+ (B_{Fdif} / F_a)^2$	0.08
$+ (B_{Fclam} / F_a)^2$	0.004
$+ (B_{Fbase} / F_a)^2$	0.0000003
$+ (\rho_o B_{Qo} / W_a)^2$	74.
$+ (Q_o B_{\rho o} / W_a)^2$	74.
$+ (\rho_f B_{Qf} / W_a)^2$	2.0
$+ (Q_f B_{\rho f} / W_a)^2$	2.0
$+ (B_{Worp} / W_a)^2$	0.0002
$+ (B_{Wfrp} / W_a)^2$	0.00005

**FIGURE 3: SPECIFIC IMPULSE BIAS LIMIT FOR  
TEST A2-542 WITH BALLPARK B's ( $B_i = 0.33\%$  or 1.5 sec)**

$(B_i / I)^2 =$	<u>Value (<math>\times 10^{-6}</math>)</u>	
$(B_{Fic} / F_a)^2$	0.667	B=0.08%
$+ (B_{Fzs} / F_a)^2$	0.004	
$+ (B_{Fpw} / F_a)^2$	0.000002	
$+ (B_{Fo} / F_a)^2$	0	
$+ (B_{Ff} / F_a)^2$	0.003	
$+ (B_{Frb} / F_a)^2$	0.00004	
$+ (B_{Fundif} / F_a)^2$	0.75	B=2.0%
$+ (B_{Fdif} / F_a)^2$	0.30	B=2.0%
$+ (B_{Fclam} / F_a)^2$	0.004	
$+ (B_{Fbase} / F_a)^2$	0.0000003	
$+ (\rho_o B_{Qo} / W_a)^2$	4.6	B=0.25%
$+ (Q_o B_{\rho o} / W_a)^2$	4.6	B=0.25%
$+ (\rho_f B_{Qf} / W_a)^2$	0.13	B=0.25%
$+ (Q_f B_{\rho f} / W_a)^2$	0.13	B=0.25%
$+ (B_{Worp} / W_a)^2$	0.0002	
$+ (B_{Wfrp} / W_a)^2$	0.00005	

The estimates of 2% for the bias limits on the correction models for the undiffused area and diffused area may be on the low side by as much as a factor of 5 or so -- this analysis shows that these uncertainty contributors are of potential importance and should be studied further.

The terms containing the bias limits in the fuel volumetric flowrate and the fuel density are not significant contributors, and the estimates of 0.25% are thought to be roughly correct. The terms containing the bias limits in the oxygen volumetric flowrate and the oxygen density are the largest contributors using the assumptions in Figure 3. The 0.25% estimate for  $B_{Q_0}$  is thought to be a good "ballpark" value. The 0.25% estimate for the bias limit for oxygen density is a good, defensible value; has potentially significant repercussions; and thus deserves further discussion.

This estimate of the bias limit for oxygen density is related to the degree of agreement between the original data sets and the curvefit(s) of those data from which the density tables were constructed as functions of temperature and pressure -- it does not consider the effects of bias errors in the measurements of oxygen temperature and pressure at the test stand flowmeter. Examination of an article in the Journal of Research of the NBS (Reference 4), which discusses agreement of the curvefits with NBS data and other previously reported data, shows that bias limit estimates of 0.25% up to 0.5% are reasonable for densities of compressed liquid and saturated liquid oxygen. Using this estimate range of 0.25% - 0.50% for the oxygen density bias limit and assuming that the bias errors are zero in every one of the other 15 variables, a range for specific impulse bias limit is calculated as 0.21% - 0.43%

or about 1 to 2 seconds. It therefore appears that the fixed errors in the oxygen density tables impose a minimum bias limit of 1 to 2 seconds for specific impulse determinations, and this certainly must be considered when deciding whether measurement systems for the other variables need improvement.

### 2.2.2 Specific impulse precision limit estimation

Since multiple SSME ground tests have been run, the existing data base of specific impulse determinations can be used with Equations (2) and (3) to obtain estimates of the precision limit(s). An unpublished analysis presented by Rocketdyne personnel at an SSME Performance Review in July 1990 presented the following SSME Phase II statistics:

$$(S_p)_{\text{test-test}} = 0.46 \text{ sec} \approx 0.5 \text{ sec}$$

$$(S_p)_{\text{engine-engine}} = 0.86 \text{ sec} \approx 1 \text{ sec}$$

If we use  $t = 2$ , the precision limits are

$$(P_p)_{\text{test-test}} \approx 1 \text{ sec}$$

$$(P_p)_{\text{engine-engine}} \approx 2 \text{ sec}$$

### 2.2.3 Specific impulse overall uncertainty estimation

Combining the specific impulse bias limit estimates and precision limit estimates as in Equation (1) we obtain a range of overall uncertainty as

$$U_I \approx 1.5 \text{ sec} \rightarrow 3 \text{ sec}$$

### 3.0 CONCLUSIONS AND RECOMMENDATIONS

In planning and designing tests and in interpreting the results of tests, the bias and precision components of experimental uncertainty should be considered separately. While consideration of only the overall uncertainty is better than not considering the effects of experimental errors at all, it does not allow one to make the most effective application of resources in experimental programs or to fully interpret the implications of the results.

The effects of correlated bias errors should be considered when comparing results from tests which have the same or some of the same error sources. Such cases occur, for example, in comparing tests on the same engine at different conditions, on the "same" engine before and after modifications, and on different engines tested on the same stand. Assuming that "the effects of all of the fixed errors subtract out" when interpreting test comparisons is, in general, incorrect.

The magnitude of the precision errors -- the "scatter" in results test-to-test and/or engine-to-engine -- should be considered when planning a test program to determine the effects of design changes. When the anticipated effect of the design change is of the same order as the precision error effects, further consideration of the test plan or test design is indicated before committing resources to the execution of the test.

The initial uncertainty analysis of the specific impulse determination in SSME tests on Stand A-2 at SSC yielded some interesting insights. The likely



magnitude of the fixed errors in the oxygen density tables sets a minimum bias limit for the specific impulse result in the  $\pm 1$  to  $\pm 2$  seconds range. More detailed analyses of the uncertainties associated with the load cells, the oxygen volumetric flowmeter, and the undiffused/diffused area model for correction to vacuum conditions are indicated since these have been identified as possible significant sources of uncertainty. The statistics of the SSME Phase II ground test data indicate a test-to-test precision limit for specific impulse of about  $\pm 1$  second and an engine-to-engine precision limit of about  $\pm 2$  seconds. This means that one would expect about 95% of the SSME specific impulse determinations on SSC Stand A-2 to fall within a band about 4 seconds wide if the engines are the "same" as those previously tested.

It is recommended that a more detailed analysis of the uncertainties mentioned in the previous paragraph be made for the SSME ground tests and that an uncertainty analysis be performed on the determination of specific impulse from Shuttle flight data so that ground test and flight results can be properly compared. The uncertainties associated with the TTB experimental program should be estimated, with particular attention to the effects of correlated bias errors, the minimum test-to-test scatter that can be expected, and the influence of experimental uncertainties on test planning with Taguchi methods. In addition, uncertainty analysis techniques should be used to analyze SRM ground testing and estimates made of the uncertainties in the specific impulse results from those tests.

#### 4.0 REFERENCES

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